# Fast Double-coupled Nonnegative Tensor Decomposition

Xiulin Wang[1,2], Tapani Ristaniemi[2], Fengyu Cong[1,2]

[1] School of Biomedical Engineering, Dalian University of Technology, China

[2] Faculty of Information Technology, University of Jyväskylä, Finland

## Introduction

***background***

- Coupled tensor decomposition has become a popular technique for the simultaneous analysis of multi-block tensors [1].
- Simultaneous extraction of common components and individual components.
- It is reasonable to expect identical elicited information among subjects since ongoing EEG are collected under the same stimulus.
- Time consumption would go extremely heavy due to the high-dimensional and non-negative nature of ongoing EEG.

***Objective***

To develop an efficient data-driven coupled tensor decomposition algorithm.

## Proposed algorithm

***Coupled tensor decomposition model***

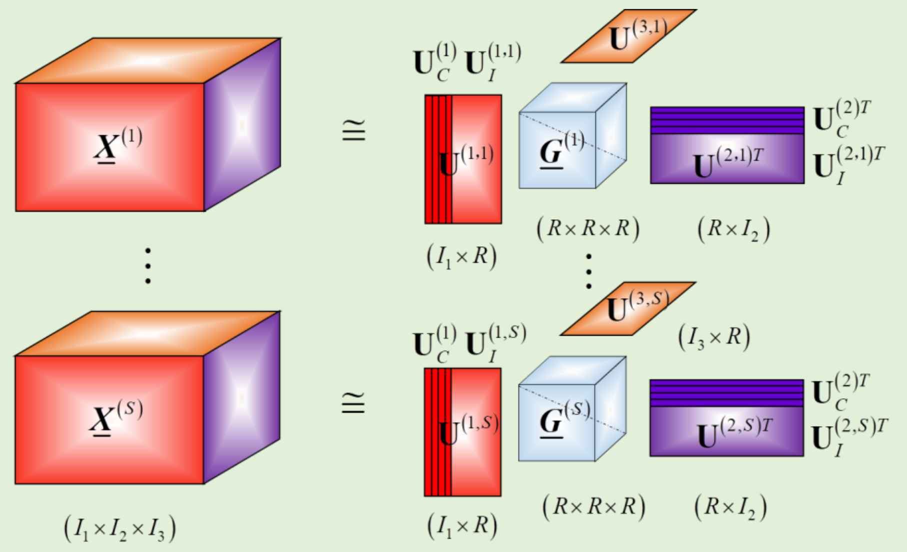

Fig.1 Conceptual illustration of dual-coupled LCPTD model [2]

***Realization of FDC-NCPD***

- Squared Euclidean Divergence minimization
- Fast Hierarchical Alternating Least Squares (Fast HALS [3])
- The object function can be expressed as:

$$\text{minimize} \sum_{s=1}^{S} \left\| \underline{\boldsymbol{X}}^{(s)} - \sum_{r=1}^{R} \lambda_r^{(s)} \boldsymbol{u}_r^{(1,s)} \circ \boldsymbol{u}_r^{(2,s)} \circ \cdots \circ \boldsymbol{u}_r^{(N,s)} \right\|_F^2$$

$$s.t.\ \boldsymbol{u}_r^{(n,1)} = \cdots = \boldsymbol{u}_r^{(n,S)}\ for\ r \leq L_n,$$

$$\left\| \boldsymbol{u}_r^{(n,s)} \right\| = 1, n = 1 \cdots N, r = 1 \cdots R\ , s = 1 \cdots S$$

## Experiments and Results

***Exp1. Validation of synthetic data***

- NTF-HALS, NTF-FastHALS, LCPTD-HALS and FDC-NCPD
- Convergence speed: Execution time, 30 runs

  SNR = 20 dB, $I_{1,2,3} = \{7n, 8n, 9n\}$, $R = 4n$, $L_{1,2} = 2n$, $S = 10$
- Decomposition quality: Performance Index, 20 runs

  SNR = -5~20 dB, $I_{1,2,3} = \{40,50,60\}$, $R = 30$, $L_{1,2} = 20$, $S = 10$

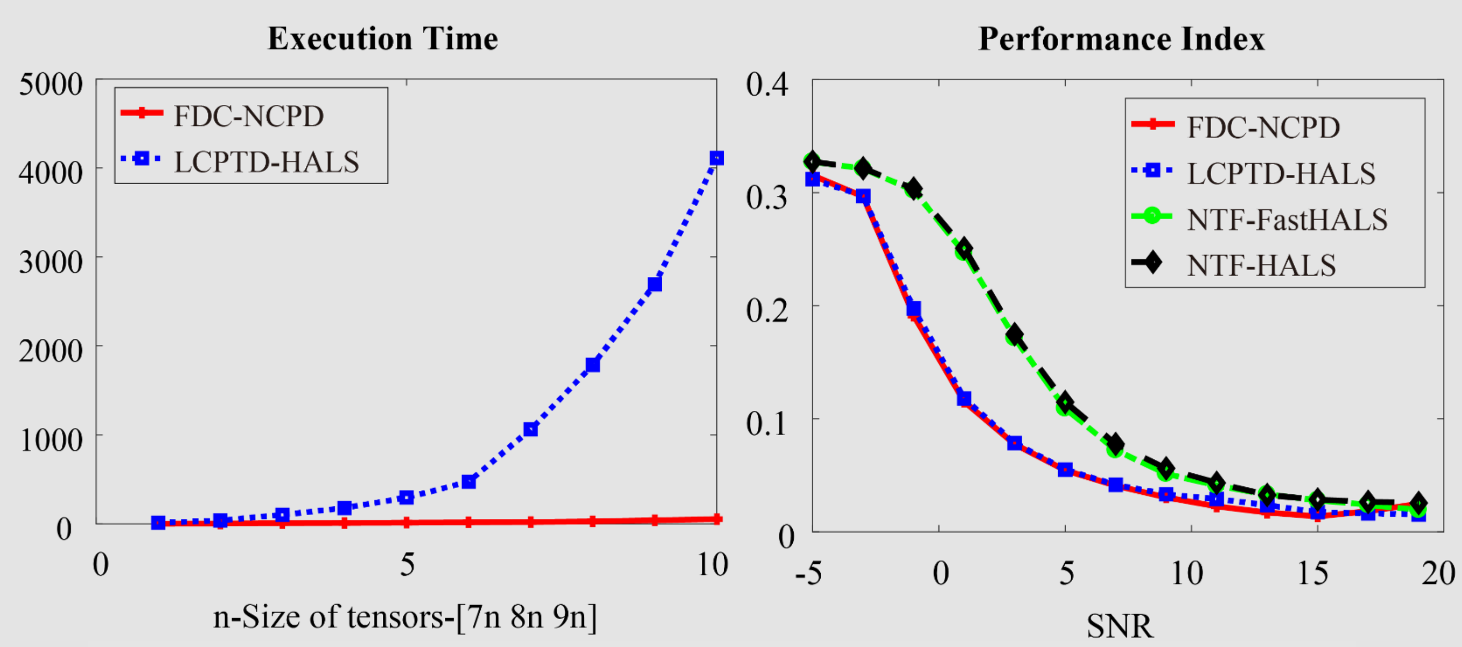


Fig 2. Results of Synthetic data

***Exp2. Application of multi-subject ongoing EEG data***

- Data collection, data preprocessing can be found in [4]
- Tensors (14): 64 channels × 146 frequency bins × 510 samples
- Coupling information exists on the first two modes.
- DIFFIT suggested $R = 36$. $L_{1,2} = 20$
- Running time : LCPTD-HALS - **76442.65** s ; FDC-NCPD - **350.97** s
- Tensor fitting : LCPTD-HALS - **0.7360** ; FDC-NCPD - **0.7353**

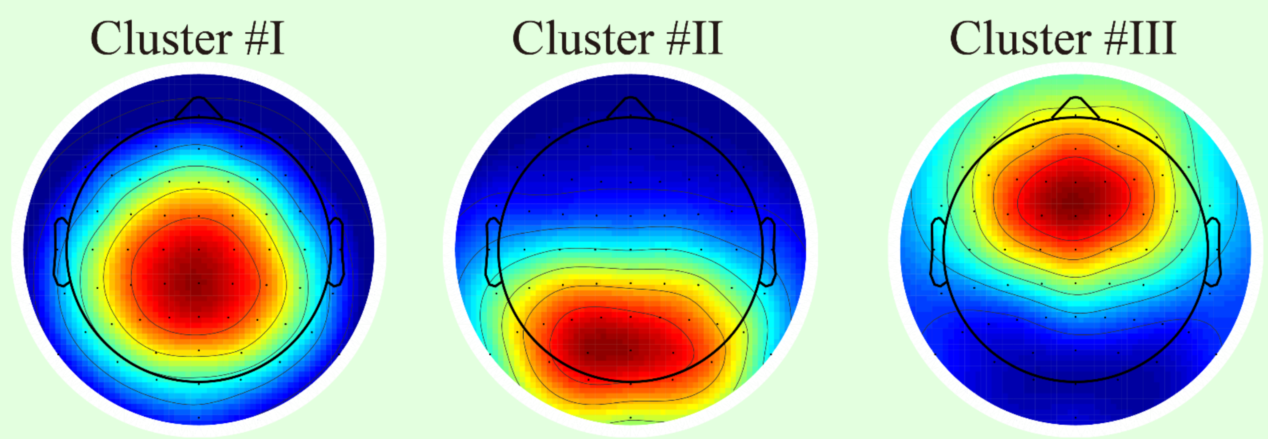


Fig 3. Averaged topographies of interest clusters from 10 runs.

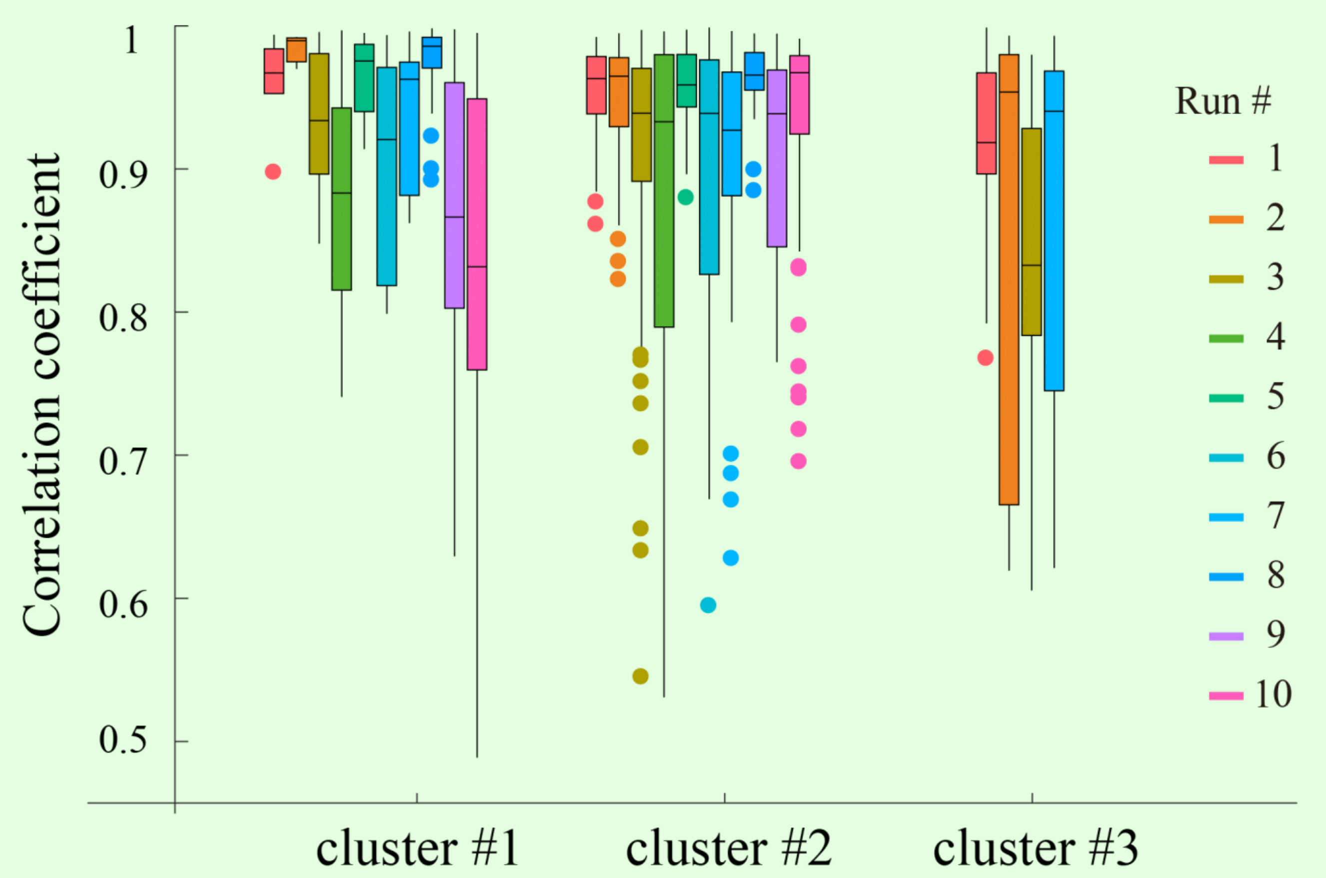


Fig 4. Correlation coefficients of internal components of clusters in 10 runs

## Conclusion

Double-coupled nonnegative tensor decomposition algorithm based on LCPTD model and Fast-HALS strategy greatly reduces the computational complexity without compromising the decomposition quality.

## References

[1] G. X. Zhou et al., "Linked component analysis from matrices to high-order tensors: Applications to biomedical data," Proceedings of the IEEE, 2016.

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[4] F. Y. Cong et al., "Linking brain responses to naturalistic music through analysis of ongoing eeg and stimulus features," IEEE Transactions on Multimedia, 2013.

## Contact

Xiulin Wang, xiulin.wang@foxmail.com;
Tapani Ristaniemi, tapani.e.ristaniemi@jyu.fi;
Fengyu Cong, cong@dlut.edu.cn;

http://xiulin.wang

JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

大连理工大学
DALIAN UNIVERSITY OF TECHNOLOGY